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LETTER TO THE EDITOR

Extended Regge trajectories

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Abstract. It is proposed that in the classification of elementary particles (with mass) a quantum number j plays a role, where $j+\frac{1}{2}=\epsilon(J+\frac{1}{2})$, with $\epsilon=\pm1$, J being the normal spin. This systematises the data concerning Regge trajectories, pairs of trajectories now uniting into single straight extended trajectories, where j is plotted against ϵm^2. For baryons, ϵ is found to be the 'naturality' of the parity, while for mesons, passing from $\epsilon=1$ to $\epsilon=-1$ leaves the parity unaltered.

In this note we point out a certain rather remarkable but simple property that appears to be exhibited by the data concerning Regge trajectories, and which does not seem to have been noted before. We are also proposing a somewhat unconventional way of interpreting this observation, although our primary concern, here, is not with theory (which we hope to discuss elsewhere) but with the data itself.

In conventional terms, our observation can be stated simply in the following way. Using a standard Chew–Frautschi plot, with J (the spin) plotted against m^2, we find very closely, that for the baryons, for each of the cases N, Δ Λ, Σ, Ξ separately, the leading natural parity trajectory is not only straight and parallel to the leading unnatural parity trajectory, but is such that *the two trajectories* (nearly coincident in the case of N and Λ) *intersect the J-axis in two points whose mid-point is* $J=-\frac{1}{2}$. In the case of the mesons, the statement is essentially the same but where now both trajectories must have the *same* naturality and we refer, instead, to the leading trajectory and a daughter.

This general property of intercepts appears to have no obvious explanation from standard Regge theory, but we had anticipated that something of the sort should hold on the basis of the unconventional proposal that we are putting forward. This, in effect, is that in the classification of hadron states there is a (new) quantum number j, which can take both positive and *negative* integral or half-integral values and for which $j(j+1)=\mathcal{J}^2=J(J+1)$(so that $j+\frac{1}{2}=\pm(J+\frac{1}{2})$, units with $\hbar=1$), J being the usual (non-negative) spin quantum number. This follows an earlier analogous proposal (Sparling 1977) which relates, more particularly, to the classification of leptons. The idea springs partly from a mathematical result (Bott 1957) concerning the representations of compact Lie groups: two such representations may be, in a sense, *different* (e.g., here, the j and $-j-1$ representations for the rotation group) even though they are, in the conventional sense, equivalent. We had been initially led to consider this idea while pursuing the implications of a twistor description of elementary particles (Perjés and Sparling 1976, Perjés 1975, 1977, Penrose 1975, 1977) but the idea has wide implications that are quite independent of twistor theory. Here we point out how the proposal fits in with the

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experimental data concerning Regge trajectories in a surprising way. The above-mentioned observation takes the following form: each leading trajectory joins very accurately on to a second trajectory (or to a different parity part of the 'same' line, in the case of N and Λ), plotted in the reverse direction, where the value of $j+\frac{1}{2}$ is deemed to have opposite signs for the two portions of what we now consider to be, in each case a *single (remarkably straight) extended Regge trajectory.*

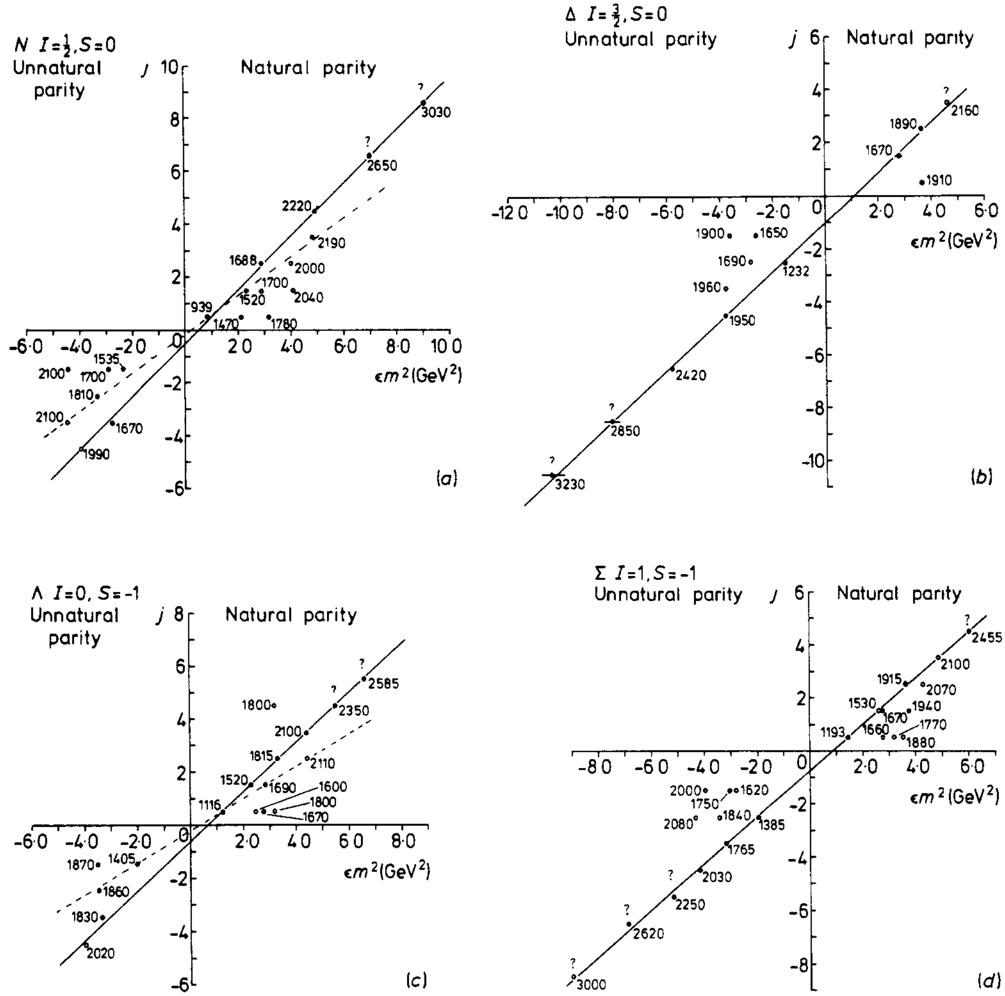
To exhibit this feature we require a slight modification of the usual Chew–Frautschi plot (Collins (1977) and bibliography therein, Chan Hong-Mo and Tsou Sheung Tsun (1978) for an up-to-date review), where now j is plotted against ϵm^2, with $\epsilon = \text{sgn}(j+\frac{1}{2})$, so that $j+\frac{1}{2} = \epsilon(J+\frac{1}{2})$. Introduce ν to represent the 'naturality' of the parity, defined according to $\nu = P(-1)^{J-\frac{1}{2}B}$ (where P denotes the parity and B the baryon number). We find that $\epsilon = \nu$ in the case of baryons, yielding a striking straightness for the extended trajectories in each case (figure 1). For the mesons, all particles on the same extended trajectory turn out to have the *same* naturality ν. Thus we have no independent means of determining ϵ (and consequently j) for mesons, but find that ϵ can be assigned opposite values for the leading trajectory and one of the 'daughter' trajectories, the two combining to give a remarkably straight extended trajectory in each of the five cases where well-established 'daughters' are seen (figure 2).

One significant consequence of these plots, if our point of view is accepted, is that they can provide a good interpolated determination of the value for the intercept with $m^2 = 0$, this being theoretically important for the high-energy behaviour of scattering cross sections (Collins 1977 and Chan Hong-Mo and Tsou Sheung Tsun 1978). We should emphasise, however, that we do not necessarily anticipate that all interpolated points at the central part of the extended trajectory should be occupied (and we point out that $j = -\frac{1}{2}$ must always be *un*occupied since the corresponding number spin states is $|2j+1| = 0$). We make no suggestions as to what 'nonsense' mechanisms might be operative in our scheme to suppress resonances generally.

Since the role played by daughter trajectories now seems different from what it had been in the conventional viewpoint we may ask how other possible daughter trajectories are to be treated. We would have difficulty in fitting in many parallel daughters because of the Froissart bound, according to which the extended trajectory's intercept with $m^2 = 0$ should presumably now lie between $j = -2$ and $j = +1$. But points of a lattice can always be joined up in different ways, and we are led to entertain a tentative suggestion that there might be extended daughter trajectories with shallower slopes from those of the leading extended trajectory. In fact we find, in some cases, that straight lines can indeed be drawn through the *ground state point* to line up several daughter points into such shallower extended trajectories. There may well be other ways of interpreting, within our scheme, the various scattered points that do not lie on the main trajectories.

We make no specific proposal concerning the new families of particles (Dalitz *et al* 1977, Herb *et al* 1977), except to say that we would anticipate j playing a role here, also.

Finally, we point out that there are many predictive ways that our idea can be used. In particular, we can make some clear-cut predictions concerning the spin-parity assignments to resonances. For the N^* and Λ, since the conventional natural and unnatural partity trajectories show degeneracy with an intercept around $-\frac{1}{2}$, our predictions are the same as those of ordinary linearly rising Regge trajectories. For the ot er trajectories, however, our predictions on spin-parity will be non-trivial. For instance, we would assign $J^P = \frac{15}{2}^+, \frac{19}{2}^+$ to the $\Delta(2850)$ and $\Delta(3230)$ respectively. As another example, we would predict the existence of an $I = \frac{1}{2}$ (unnatural parity) straight



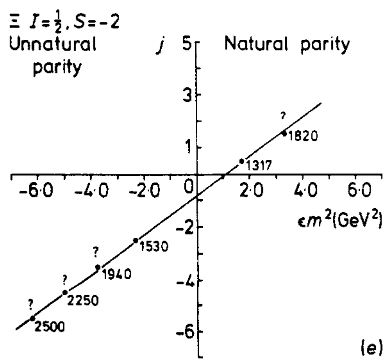


Figure 1. Modified Chew–Frautschi plot for the baryon resonances, according to the Review of Particle Properties (1976). Solid circles represent states with three or four star rating, and open circles states with one or two star rating. A question mark indicates that the spin-parity assignment of the resonance is tentative.

Regge trajectory corresponding to the extension into the negative region of the line of figure 2(f). This would mean, in the ordinary way of plotting things, a 'daughter' line parallel to KL and displaced downwards by a J value of approximately $\frac{1}{2}$.

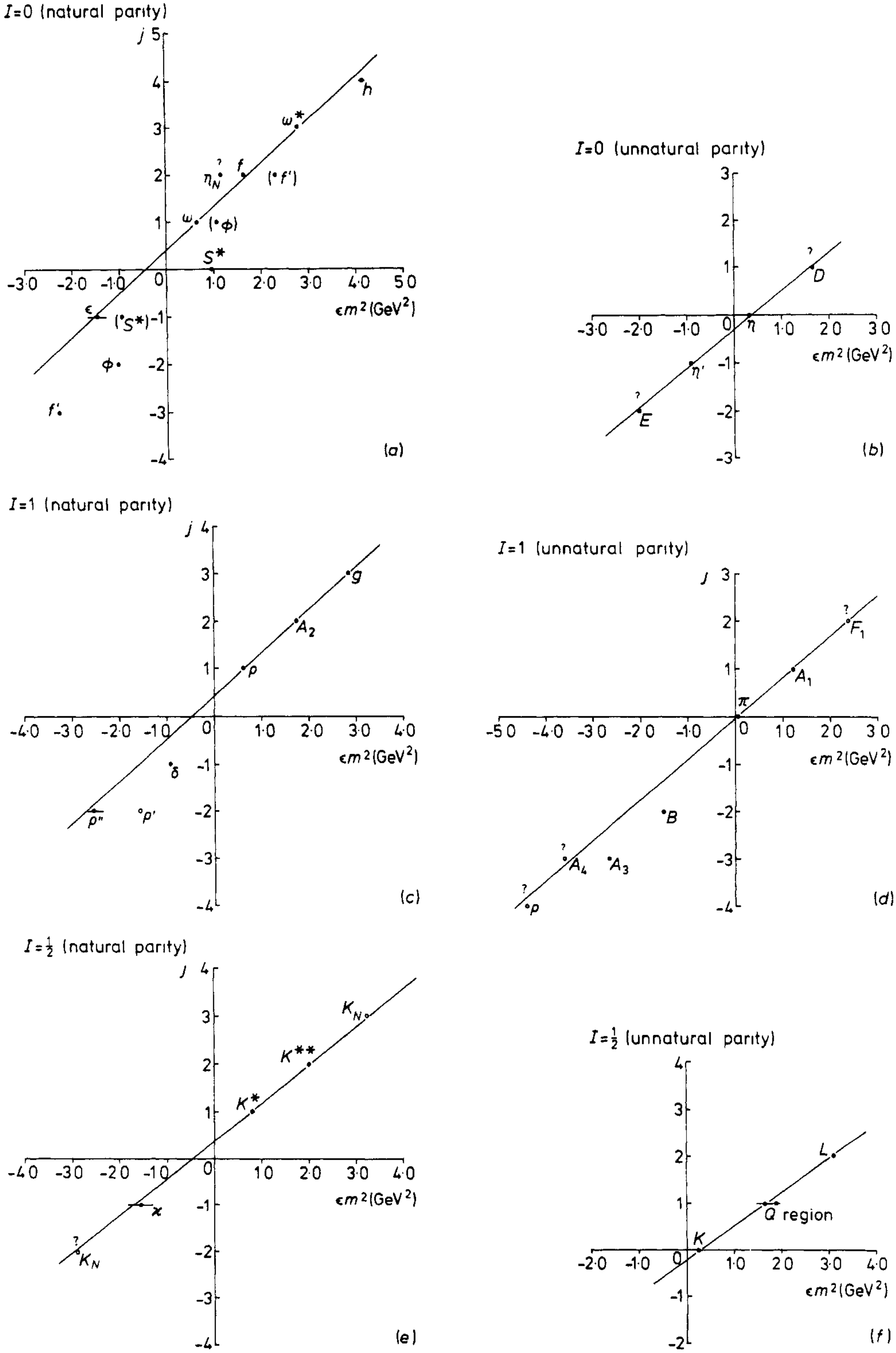


Figure 2. Modified Chew–Frautschi plot for the meson resonances, according to the Review of Particle Properties (1976). Solid circles represent states included in the Meson Table, and open circles left out from it. A question mark indicates that the spin-parity assignment of the resonance is tentative. In (*a*) brackets are put around alternative positions of resonances already marked elsewhere corresponding to a different value of ϵ.

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